

PERTEMUAN 7

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Rumus – rumus turunan fungsi eksponen

$$(1) f(x) = e^x \Rightarrow f'(x) = e^x$$

$$(2) f(x) = e^{gx} \Rightarrow f'(x) = e^{gx} \cdot g'(x)$$

$$(3) f(x) = a^x \Rightarrow f'(x) = a^x \ln a, \quad a > 0 ; a \neq 1$$

$$(4) f(x) = a^{g(x)} \Rightarrow f'(x) = a^{g(x)} \cdot g'(x); \quad a > 0 \text{ dan } a \neq 1$$

Sifat – sifat Logaritma Natural

(1) $e^{\ln a} = a$

(2) $\ln ab = \ln a + \ln b$

(3) $\ln \frac{a}{b} = \ln a - \ln b$

(4) $\ln a^n = n \cdot \ln a$

(5) $\ln e^n = n$ [$\cdot \ln e = 1$]

Rumus – rumus Turunan Fungsi Logaritma

(1) $y = \log_a x \Rightarrow y' = \frac{1}{x \log a}$

(2) $y = \log_a g(x) \Rightarrow y' = \frac{g'(x)}{g(x) \log a}$

(3) $y = \ln x \Rightarrow y' = \frac{1}{x}$

(4) $y = \ln g(x) \Rightarrow y' = \frac{g'(x)}{g(x)}$

Rumus Turunan Fungsi Trigonometri

1. $y = \sin x \rightarrow y' = \cos x$
2. $y = \cos x \rightarrow y' = -\sin x$
3. $y = \tan x \rightarrow y' = \sec^2 x$
4. $y = \cot x \rightarrow y' = -\csc^2 x$
5. $y = \sec x \rightarrow y'$
6. $y = \csc x \rightarrow -\csc \times \cot x$
7. $y = \sin^n x \rightarrow y' = n \sin^{n-1} x \times \cos x$
8. $y = \cos^n x \rightarrow y' = -n \cos^{n-1} x \times \sin x$
9. $y = \sin u \rightarrow y' = u' \cos u$
10. $y = \cos u \rightarrow y' = -u' \sin u$
11. $y = \tan u \rightarrow y' = u' \sec^2 u$
12. $y = \cot u \rightarrow y' = -u' \csc^2 u$
13. $y = \sec u \rightarrow y' = u' \sec u \tan u$
14. $y = \csc u \rightarrow y' = -u' \csc u \cot u$
15. $y = \sin^n u \rightarrow y' = n \cdot u' \sin^{n-1} u \cos u$
16. $y = \cos^n u \rightarrow y' = -n \cdot u' \cos^{n-1} u \cdot \sin u$

Contoh Soal

$$1. y = \sin 2x \text{ maka } y' = 2 \cos 2x$$

$$2. y = \cos (3x + 1) \text{ maka } y' = -3 \sin (3x + 1)$$

$$3. y = \sin (3x - 2) \cos 2x$$

Jawab

$$y = \sin (3x - 2) \cos 2x$$

$$U = \sin (3x - 2) \text{ maka } U' = 3 \cos (3x - 2)$$

$$V = \cos 2x \text{ maka } V' = -2 \sin 2x$$

$$y' = U'V + V'U$$

$$y' = 3 \cos (3x - 2)(\cos 2x) + (-2 \sin 2x)(\sin (3x - 2))$$

$$4. y = \frac{\sin x}{\tan x}$$

Jawab

$$y = \frac{\sin x}{\tan x}$$

$$U = \sin x \text{ maka } U' = \cos x$$

$$V = \tan x \text{ maka } V' = \sec^2 x$$

$$y' = \frac{U'V - V'U}{V^2}$$

$$y' = \frac{\cos x \tan x - \sec^2 x \sin x}{\tan^2 x}$$

$$5. y = {}^2 \log x$$

$$y' = \frac{1}{x} {}^2 \log e$$

Contoh Soal

$$y = \frac{\sin x}{\sin x + \cos x}$$

Menentukan y'

Misal :

$$u = \sin x, \text{ maka } u' = \cos x$$

$$v = \sin x + \cos x, \text{ maka } v' = \cos x - \sin x$$

Jika $y = \frac{u}{v}$, maka:

$$y' = \frac{u'v - uv'}{v^2}$$

$$= \frac{(\cos x)(\sin x + \cos x) - (\sin x)(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\sin x \cos x + \cos^2 x - \sin x \cos x + \sin^2 x}{(\sin x + \cos x)^2}$$

$$= \frac{1}{(\sin x + \cos x)^2}$$

Latihan Soal

$$y = {}^2\text{Log}(2x^3 - x^2 + x - 5)$$

$$y = {}^{(2x+1)}\text{Log}(x - 2)$$

$$y = \ln(x^2 - 3x + 2)$$

$$y = 3^{3x^2 - 2x + 1}$$

$$y = e^{3x^2 - 2x + 1}$$

$$y = \text{Sin } 2x \text{ Cos } 5x$$

$$y = \text{Tan } 2x \text{ Cos } (5x - 2)$$

$$y = \frac{\text{Cos } x}{\text{Sin } x}$$

$$y = \frac{\text{Cos } (2x - 1)}{\text{Tan } x}$$

$$y = \frac{\text{Sec } x}{\text{Cos } x}$$