

PERTEMUAN 11

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A. Integral Trigonometri

1.	$\int \sin x \, dx = -\cos x + C$	12.	$\int \csc ax \, dx = \frac{1}{a} \ln \csc ax - \cot ax + C$
2.	$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$	13.	$\int \sec^2 x \, dx = \tan x + C$
3.	$\int \cos x \, dx = \sin x + C$	14.	$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$
4.	$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$	15.	$\int \csc^2 x \, dx = -\cot x + C$
5.	$\int \tan x \, dx = \ln \sec x + C = -\ln \cos x + C$	16.	$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$
6.	$\int \tan ax \, dx = \frac{1}{a} \ln \sec ax + C = -\frac{1}{a} \ln \cos ax + C$	17.	$\int \sec x \tan x \, dx = \sec x + C$
7.	$\int \cot x \, dx = \ln \sin x + C$	18.	$\int \csc x \cot x \, dx = -\csc x + C$
8.	$\int \cot ax \, dx = \frac{1}{a} \ln \sin ax + C$	19.	$\int \cot^2 x \, dx = -\cot x - x + C$
9.	$\int \sec x \, dx = \ln \sec x + \tan x + C$	20.	$\int \sin^2 x \, dx = \frac{1}{2} x - \frac{1}{4} \sin 2x + C$
10.	$\int \sec ax \, dx = \frac{1}{a} \ln \sec ax + \tan ax + C$	21.	$\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{4} \sin 2x + C$
11.	$\int \csc x \, dx = \ln \csc x - \cot x + C$	22.	$\int \tan^2 x \, dx = \tan x - x + C$

B. Integral Eksponen dan Logaritma

1.	$\int e^u du = e^u + C$ atau $\int e^x dx = e^x + C$
2.	$\int a^x dx = \left(\frac{1}{\ln a}\right) a^x + C$, dimana $a \neq 1$
3.	$\int \frac{1}{x} dx = \ln x + C$
4.	$\int \frac{1}{ax} dx = \frac{1}{a} \int \frac{1}{x} dx = \frac{1}{a} \ln x + C$, dimana $a \neq 0$
5.	$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$, dimana $a \neq 0$

Contoh:

$$1. \int \sin 4x \, dx = -\frac{1}{4} \cos 4x + c$$

$$2. \int (3 \sin t - 2 \cos t) dx = -3 \cos t - 2 \sin t + c$$

$$3. \int (\sin 10x + 5 \cos 7x + \tan 2x)$$

$$= -\frac{1}{10} \cos 10x + \frac{5}{7} \sin 7x - \frac{1}{2} \ln |\cos 2x| + c$$

$$4. \int \cos (2x - 1) \, dx = \frac{1}{2} \sin(2x - 1) + c$$

$$5. \int (\sin^2 x + \tan^2 x + 5 + \cos^2 x) dx$$

$$= \int (1 + \tan^2 x + 5) dx = \int (\sec^2 x + 5) dx = \tan x + 5x + c$$

$$6. \int \left(\sec \frac{1}{3} x \cdot \tan \frac{1}{3} x - \left(\frac{2}{\sqrt{9-x^2}} \right) + \operatorname{cosec}^2 2x \right) dx$$

$$= \int \left(\sec \frac{1}{3} x \cdot \tan \frac{1}{3} x \right) dx - \int \left(\frac{2}{\sqrt{9-x^2}} \right) dx + \int (\operatorname{cosec}^2 2x) dx$$

$$= 3 \sec \frac{1}{3} x - 2 \operatorname{arc} \sin \left(\frac{x}{3} \right) - \frac{1}{2} \cot 2x + c$$

$$\begin{aligned}
7. \quad \int \cos^5 x \, dx &= \int (\cos^4 x \cdot \cos x) \, dx \\
&= \int (\cos^2 x)^2 \cdot d(\sin x) = \int (1 - \sin^2 x)^2 \cdot d(\sin x) \\
&= \int (1 - 2\sin^2 x + \sin^4 x) \cdot d(\sin x) \\
&= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c
\end{aligned}$$

$$\begin{aligned}
8. \quad \int (7 \sin 4x \cos 4x) \, dx &= 7 \int (\sin 4x \cos 4x) \, dx \\
&= 7 \int 2 \sin 8x \, dx \\
&= 14 \int \sin 8x \, dx \\
&= -14 \left(\frac{1}{8} \right) \cos 8x + c \\
&= -\frac{7}{4} \cos 8x + c
\end{aligned}$$

C. Latihan Soal

1.	$\int [\sin 10x + 5 \cos x + \tan 2x] dx$
2.	$\int [7 \sin 8x + \frac{3}{5} \cos 5x + 2 \tan 3x] dx$
3.	$\int \sin x + \cos^2 x dx$
4.	$\int [\sin^2 x + \cos^2 x + \tan^2 x + 6] dx$
5.	$\int \sin 5x \cos 2x dx$
6.	$\int (\sin x - \cos x)^2 dx$
7.	$\int 1 - \sin 2x dx$
8.	$\int \cos^4 x dx$
9.	$\int \sin x - \cos x dx$
10.	$\int \sin^2 x \cos^4 x dx$

11.	$\int e^x dx$
12.	$\int e^{5x} dx$
13.	$\int (e^{\cos y} - \sin y) dy$
14.	$\int y e^{-y^2} dy$
15.	$\int (8^{\cot 2y} \csc^2 2y) dy$
16.	$\int (\frac{1}{5^x} + e^{7x} + 5) dx$
17.	$\int (\frac{8}{7x} + \frac{2}{5} e^{10x} + \frac{5}{7}) dx$
18.	$\int (8^x + \frac{2^{x+1}}{7^x}) dx$
19.	$\int (\frac{5^{x+2}}{8^{x+1}}) dx$
20.	$\int (\frac{3^{2x} + 5^x}{2^x}) dx$

➤ **Integral fungsi eksponen dan fungsi logaritma**

1. $\int \frac{dx}{x} = |\ln x| + c$
2. $\int \left(\frac{1}{ax}\right) dx = \frac{1}{a} \int \frac{dx}{x} = \frac{1}{a} |\ln x| + c$ dimana $a \neq 0$
3. $\int a^x dx = \left(\frac{1}{\ln a}\right) a^x + c$ dimana $a \neq 1, a \neq 0$
4. $\int e^x dx = e^x + c$
5. $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ dimana $a \neq 0$

Contoh:

1. $\int 2e^{5x} dx = 2 \int e^{5x} dx = \frac{2}{5} e^{5x} + c$
2. $\int \left(\frac{2}{3x} + 4^x - 1\right) dx = \frac{2}{3} \int \frac{1}{x} dx + \int 4^x dx - \int 1 dx$
 $= \frac{2}{3} \ln x + \left(\frac{1}{\ln 4}\right) 4^x - x + c$
3. $\int \left(\frac{2^{x+3}}{5^{2x+2}}\right) dx = \int \left(\frac{2^x \cdot 2^3}{5^{2x} \cdot 5^2}\right) dx = \int \left(\frac{2^3}{5^2}\right) \left(\frac{2^x}{25^x}\right) dx$
 $= \frac{8}{25} \int \left(\frac{2}{25}\right)^x = \frac{8}{25} \left(\frac{1}{\ln \frac{2}{25}}\right) \left(\frac{2}{25}\right)^x + c$
4. $\int \left(\frac{3^{2x+5^x}}{2^x}\right) dx = \int \frac{9^x}{2^x} dx + \int \frac{5^x}{2^x} dx$
 $= \int \left(\frac{9}{2}\right)^x dx + \int \left(\frac{5}{2}\right)^x dx$
 $= \left(\frac{1}{\ln \frac{9}{2}}\right) \left(\frac{9}{2}\right)^x + \left(\frac{1}{\ln \frac{5}{2}}\right) \left(\frac{5}{2}\right)^x + c$

Soal Latihan

$$1) \int \left[\frac{1}{5x} + e^{7x} + 5 \right] dx$$

$$2) \int \left[\frac{8}{7x} + \frac{2}{5} e^{10x} + \frac{5}{7} \right] dx$$

$$3) \int \left[8^x + \frac{2^{x+1}}{7^x} \right] dx$$

$$4) \int \left[\frac{5^{x+2}}{8^{x+1}} \right] dx$$

$$5) \int \left[\frac{3^{2x} + 5^x}{2^x} \right] dx$$

Soal Latihan

1) $\int 2x \sin x dx$

2) $\int x \cos 3x dx$

3) $\int 7xe^{\frac{1}{2}x} dx$

4) $\int 8x + 2 e^{-5x} dx$

5) $\int 2x 8^x dx$

6) $\int 10x + 1 5^x dx$

7) $\int x \tan^{-1} x dx$

8) $\int x \ln x dx$

9) $\int e^x \cos x dx$

10) $\int e^x \sin x dx$

11) $\int x^2 \sin 5x dx$

12) $\int 2x^2 e^{-x} dx$

13) $\int x^2 3^x dx$

14) $\int x^3 e^{2x} dx$

15) $\int x^3 \cos 4x dx$

16) $\int \left(\frac{\ln x}{x^2} \right) dx$

17) $\int \cos x \ln \sin x dx$

18) $\int \sin \sqrt{x} dx$

19) $\int e^{\sqrt{x}} dx$

20) $\int x^5 e^{x^2} dx$