

PERTEMUAN 12

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INTEGRAL TEKNIK SUBSTITUSI SEDERHANA

Penyelesaian dengan teknik substitusi sederhana menggunakan pemisalan U dan turunan U atau disebut du .

Contoh 1

$$\int \frac{1}{x+2} dx$$

Misal: $u = x + 2$

$$du = dx$$

sehingga:

$$\begin{aligned} \int \frac{1}{x+2} dx &= \int \frac{1}{u} du \\ &= \ln u + c \\ &= \ln(x+2) + c \end{aligned}$$

Contoh 2

$$\int \frac{x}{2x^2 + 4} dx$$

Misal: $u = 2x^2 + 4$

$$du = 4x dx$$

$$dx = \frac{du}{4x}$$

sehingga:

$$\int \frac{x}{2x^2 + 4} dx = \int \frac{x}{u} \cdot \frac{du}{4x}$$

$$= \frac{1}{4} \int \frac{du}{u}$$

$$= \frac{1}{4} \ln u + c$$

$$= \frac{1}{4} \ln(2x^2 + 4) + c$$

Contoh 3

$$\int (3x + 6)^8 dx$$

Misal: $u = 3x + 6$

$$du = 3 dx$$

$$dx = \frac{du}{3}$$

sehingga:

$$\int (3x + 6)^8 dx = \int u^8 \frac{du}{3}$$

$$= \frac{1}{3} \int u^8 du$$

$$= \frac{1}{3} \cdot \frac{1}{8+1} u^{8+1} + c$$

$$= \frac{1}{3} \cdot \frac{1}{9} \cdot u^9 + c$$

$$= \frac{1}{27} (3x + 6)^9 + c$$

Contoh 4

$$\int \frac{\sin x}{\cos x} dx$$

Misal: $u = \cos x$

$$du = -\sin x dx$$

$$dx = \frac{du}{-\sin x}$$

sehingga:

$$\int \frac{\sin x}{\cos x} dx = \int \frac{\sin x}{u} \cdot \frac{du}{-\sin x}$$

$$= - \int \frac{du}{u}$$

$$= - \ln u + c$$

$$= - \ln \cos x + c$$

Contoh 5

$$\int \sin^2 x \cos x \, dx$$

Misal: $u = \sin x$

$$du = \cos x \, dx$$

$$dx = \frac{du}{\cos x}$$

sehingga:

$$\int \sin^2 x \cos x \, dx = \int u^2 \cdot \cos x \cdot \frac{du}{\cos x}$$

$$= \int u^2 \, du$$

$$= \frac{1}{3} u^3 + C$$

$$= \frac{1}{3} \sin^3 x + C$$

$$1. \int (x^2 - 4)^3 2x \, dx$$

Jawab:

$$\text{Misal } u = x^2 - 4$$

$$du = 2x \, dx$$

$$\text{Sehingga, } \int (x^2 - 4)^3 2x \, dx = \int u^3 du = \frac{1}{4}u^4 + c = \frac{1}{4}(x^2 - 4)^4 + c$$

$$2. \int (5x^2 + 1)\sqrt{5x^3 + 3x - 2} \, dx$$

Jawab:

$$\text{Misal } u = 5x^3 + 3x - 2$$

$$du = (15x^2 + 3)dx$$

$$du = 3(5x^2 + 1)dx$$

$$\frac{1}{3}du = (5x^2 + 1)dx$$

$$\text{Jadi, } \int (5x^2 + 1)\sqrt{5x^3 + 3x - 2} \, dx = \frac{1}{3} \int \sqrt{u} \, du$$

$$= \frac{1}{3} \int u^{\frac{1}{2}} \, du = \frac{2}{9}u^{\frac{3}{2}} + c = \frac{2}{9}(5x^3 + 3x - 2)^{\frac{3}{2}} + c$$

$$3. \int \frac{x^3}{(x^4 + 5)^{\frac{1}{4}}} \, dx$$

Jawab:

$$\text{misal } u = x^4 + 5$$

$$du = d(x^4 + 5)$$

$$= 4x^3 dx$$

$$\int \frac{x^3}{(x^4 + 5)^{\frac{1}{4}}} \, dx = \int \frac{x^3}{u^{\frac{1}{4}}} \cdot \frac{du}{4x^3}$$

$$= \frac{1}{4} \int u^{-\frac{1}{4}} du = \frac{1}{3} u^{\frac{3}{4}} + c$$

$$= \frac{1}{3} (x^4 + 5)^{\frac{3}{4}} + c$$

$$4. \int \sin^4 x \cdot \cos x \, dx$$

Jawab

$$\text{Misal } u = \sin x \Rightarrow du = \cos x \, dx$$

$$\text{Jadi } \int \sin^4 x \cdot \cos x \, dx = \int u^4 \, du = \frac{1}{5}u^5 + c = \frac{1}{5} \sin^5 x + c$$

5. $\int (\sin^5 x^2) (x \cos x^2) dx$

Jawab:

$$\text{Misal } u = \sin x^2 \Rightarrow du = 2x \cos x^2 dx \Rightarrow \frac{1}{2} du = x \cos x^2 dx$$

$$\text{Jadi, } \int (\sin^5 x^2) (x \cos x^2) dx = \frac{1}{2} \int u^5 du = \frac{1}{12} u^6 = \frac{1}{12} \sin^6 x^2 + c$$

Latihan Soal

1. $\int 2x (x^2 + 1)^4 dx$

2. $\int \frac{3}{\sqrt{2x+4}} dx$

3. $\int x \sqrt{x^2 - 4} dx$

4. $\int (2x + 1) (2x^2 + 2x + 4)^5 dx$

5. $\int \frac{2x^2}{\sqrt{4x^3 + 6}} dx$

6. $\int (x^2 - 4)^3 2x dx$

7. $\int (5x^2 + 1) \sqrt{5x^3 + 3x - 2} dx$

8. $\int \frac{x^3}{(x^4 + 5)^{\frac{1}{3}}} dx$

9. $\int \sin^4 x \cos x dx$

10. $\int \frac{x^3 + x}{(x^4 + 2x^2 + 8)} dx$

□

INTEGRAL TEKNIK PARSIAL

Integral parsial merupakan salah satu teknik pengintegralan jika teknik integral yang lain tidak dapat diselesaikan seperti teknik integral substitusi atau integral tak tentu secara umum. Metode integral parsial didasarkan pada integrasi untuk turunan hasil kali dua fungsi.

Jika $u = u(x)$ dan $v = v(x)$ maka rumus integral parsial adalah:

$$d(uv) = u \cdot dv + v \cdot du$$

$$u \, dv = d(uv) - v \, du$$

$$\int u \, dv = uv - \int v \, du$$

Ada dua hal yang sangat penting dalam integral parsial dan akan menentukan berhasil atau tidaknya pengintegralan, yaitu :

Pemilihan u dan dv yang tepat, memilih dv sehingga v dapat ditentukan melalui $v = \int dv$.

$\int v \, du$ harus lebih mudah diselesaikan dibandingkan $\int u \, dv$.

Catatan :

1. Integral yang diberikan harus dipisahkan menjadi 2 bagian u , du , dx dan dv .
2. Yang dipilih dv harus yang dapat segera diintegrasi.
3. $\int v \, du$ tidak boleh lebih sulit dari pada $\int u \, dv$

Contoh 1

$$\int x \cdot \sin x \, dx$$

Misalkan : $u = x$

$$du = dx$$

$$dv = \sin x \, dx$$

$$v = \int \sin x \, dx = -\cos x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x \sin x \, dx = x \cdot (-\cos x) - \int -\cos x \, dx$$

$$= -x \cdot \cos x + \sin x + c$$

Contoh 2

$$\int x \cdot \cos x \, dx$$

Misalkan : $u = x$

$$du = dx$$

$$dv = \cos x \, dx$$

$$v = \int \cos x \, dx = \sin x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x \cdot \cos x \, dx = x \cdot \sin x - \int \sin x \, dx$$

$$= x \cdot \sin x + \cos x + c$$

Contoh 3

$$\int \sec^3 x \, dx$$

$$\int \sec^3 x \, dx = \int \sec x \sec^2 x \, dx$$

Misalkan : $u = \sec x$

$$du = \sec x \tan x \, dx$$

$$dv = \sec^2 x$$

$$v = \int \sec^2 x \, dx = \tan x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \tan x \sec x \tan x \, dx$$

$$= \sec x \tan x - \int \tan^2 x \sec x \, dx$$

$$= \sec x \tan x - \int (\sec^2 x - 1) \sec x \, dx$$

$$= \sec x \tan x - \int (\sec^3 x - \sec x) \, dx$$

$$\Rightarrow \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2} (\sec x \tan x) + \frac{1}{2} \ln|\sec x + \tan x| + C$$

1. $\int x e^x dx$

Jawab:

Misal $u = x \Rightarrow du = dx$

$$dv = e^x dx \Rightarrow v = e^x$$

$$\int x e^x dx = x e^x - \int e^x dx$$

$$= x e^x - e^x + c$$

$$= e^x(x - 1) + c$$

2. $\int x \sin 3x dx$

Jawab:

Misal $u = x \Rightarrow du = dx$

$$dv = \sin 3x dx \Rightarrow v = -\frac{1}{3} \cos 3x$$

$$\int x \sin 3x dx = -x \left(\frac{1}{3} \cos 3x \right) + \frac{1}{3} \int \cos 3x dx$$

$$= -\frac{1}{3} x \cos x + \frac{1}{3} \int \cos 3x dx$$

$$= -\frac{1}{3} x \cos x + \frac{1}{3} \cdot \frac{1}{3} \sin 3x + c$$

$$= -\frac{1}{3} \left(x \cos x - \frac{1}{3} \sin 3x \right) + c$$

3. $\int x \sqrt{x+1} dx$

Jawab :

Misal $u = x \Rightarrow du = dx$

$$dv = \sqrt{x+1} dx \Rightarrow v = \frac{2}{3} (x+1)^{\frac{3}{2}}$$

$$\int x \sqrt{x+1} dx = x \cdot \frac{2}{3} (x+1)^{\frac{3}{2}} - \frac{2}{3} \int (x+1)^{\frac{3}{2}} dx$$

$$= \frac{2}{3} x (x+1)^{\frac{3}{2}} - \frac{4}{15} (x+1)^{\frac{5}{2}} + c$$

$$4. \int e^{-x} \cos x \, dx$$

Jawab:

$$\text{Misal } u = e^{-x} \Rightarrow du = -e^{-x} dx$$

$$dv = \cos x \, dx \Rightarrow v = \sin x$$

$$\int e^{-x} \cos x \, dx = -e^{-x} \sin x - \int e^{-x} \sin x \, dx$$

$$\int e^{-x} \cos x \, dx = -e^{-x} \sin x - \left(-e^{-x} \cdot \cos x - \int -e^{-x} \cdot \cos x \, dx \right)$$

$$\int e^{-x} \cos x \, dx = -e^{-x} \sin x + e^{-x} \cdot \cos x - \int e^{-x} \cdot \cos x \, dx$$

$$2 \int e^{-x} \cos x \, dx = -e^{-x} \sin x + e^{-x} \cdot \cos x$$

$$\int e^{-x} \cos x \, dx = \frac{1}{2} (-e^{-x} \sin x + e^{-x} \cdot \cos x)$$

$$5. \int \ln x \, dx$$

Jawab:

$$\text{Misal } u = \ln x \Rightarrow du = \frac{dx}{x}$$

$$dv = dx \Rightarrow v = x$$

$$\int \ln x \, dx = x \ln x - \int x \cdot \frac{dx}{x}$$

$$= x \ln x - x$$

Latihan Soal

1. $\int x(x+3)^2 dx$

2. $\int 8x + 2e^{-5x} dx$

3. $\int 7xe^{\frac{1}{2}x} dx$

4. $\int x^2 \sin \frac{1}{2}x dx$

5. $\int 2x \sin x dx$

6. $\int x \cos 3x dx$

7. $\int e^x \cos x dx$

8. $\int x^2 \cos x dx$

9. $\int x^2 \cos(3x + \pi) dx$

10. $\int x^3 \sin x dx$

11. $\int (2x - 1) \cos 2x dx$

12. $\int x^2 \sin x dx$

13. $\int x^2 \sqrt{(4 - 2x)} dx$

14. $\int \frac{x}{\sqrt{1-x}} dx$

15. $(4x + 2) \cos \int (2x + 5) dx$

□